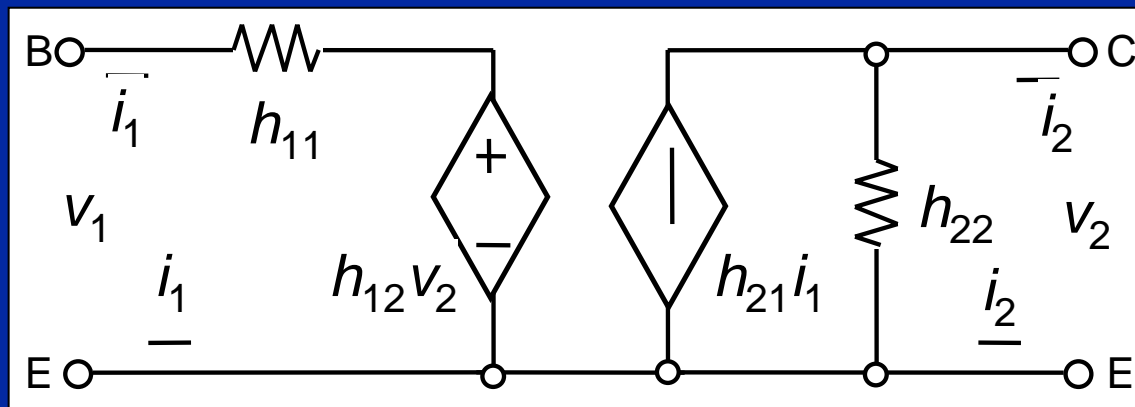


Small Signal Response

h parameter small-signal eq.circuit



$$v_1 = h_{11}i_1 + h_{12}v_2$$

$$i_2 = h_{21}i_1 + h_{22}v_2$$

Short-circuit input impedance :

$$h_{11} \equiv h_i = (v_1/i_1) \Big|_{v_2=0}$$

Open circuit reverse voltage ratio :

$$h_{12} \equiv h_r = (v_1/v_2) \Big|_{i_1=0}$$

Short-circuit forward current ratio :

$$h_{21} \equiv h_f = (i_2/i_1) \Big|_{v_2=0}$$

Open circuit output admittance :

$$h_{22} \equiv h_o = (i_2/v_2) \Big|_{i_1=0}$$

From T. A. Fjeldly, T. Ytterdal, M. S. Shur, *Introduction to Device Modeling and Circuit Simulation*, Wiley, New York, 1998

Notation for h -parameters

The parameter h_{11} (h_i) is called the **short-circuit input impedance**, h_{12} is called the **open circuit reverse voltage ratio** (h_r), h_{21} is called the **short-circuit forward current ratio** (h_f), and h_{22} is called the **open-circuit output admittance** (h_o). In the alternate notation (h_i , h_r , h_f , and h_o), a second subscript is often used to denote the transistor configuration. For example, the h -parameters for the common-emitter transistor circuit configuration are denoted as h_{ie} , h_{re} , h_{fe} , and h_{oe} . The total of h -parameters is twelve (four for each transistor configuration). In their data sheets, transistor manufacturers usually provide only common-emitter h -parameters.

h parameters

Common-emitter
h-parameters

Common-base
h-parameters

Common-collector
h-parameters

h_{ie}

$$h_{ib} = h_{ie} / (h_{fe} + 1)$$

$$h_{ic} = h_{ie}$$

h_{re}

$$h_{rb} = h_{ie} h_{oe} / (h_{fe} + 1) - h_{re}$$

$$h_{rc} = 1$$

h_{fe}

$$h_{fb} = -h_{fe} / (h_{fe} + 1)$$

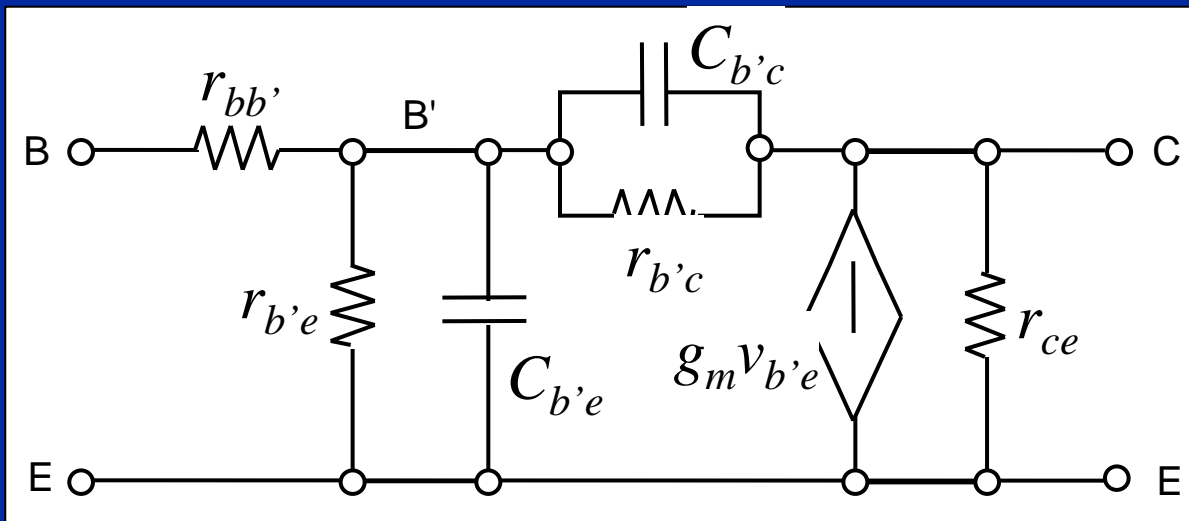
$$h_{fc} = -h_{fe} - 1$$

h_{oe}

$$h_{ob} = h_{oe} / (h_{fe} + 1)$$

$$h_{oc} = h_{oe}$$

Hybrid- π Equivalent Circuit



Often used in CE configuration.

Relates well to physical (SPICE) parameters.

$$g_m = \frac{\partial I_c}{\partial V_{b'e}} \approx \frac{I_c}{V_{th}}$$

$$r_{b'e} = \frac{\partial V_{b'e}}{\partial I_b} = \frac{\partial V_{b'e}}{\partial I_c / \beta_N} = \frac{\beta_N}{g_m}$$

$$r_{ce} = \frac{\partial V_{ce}}{\partial I_c} \approx \frac{|V_A|}{I_c} \approx \frac{|V_A|}{g_m V_{th}}$$

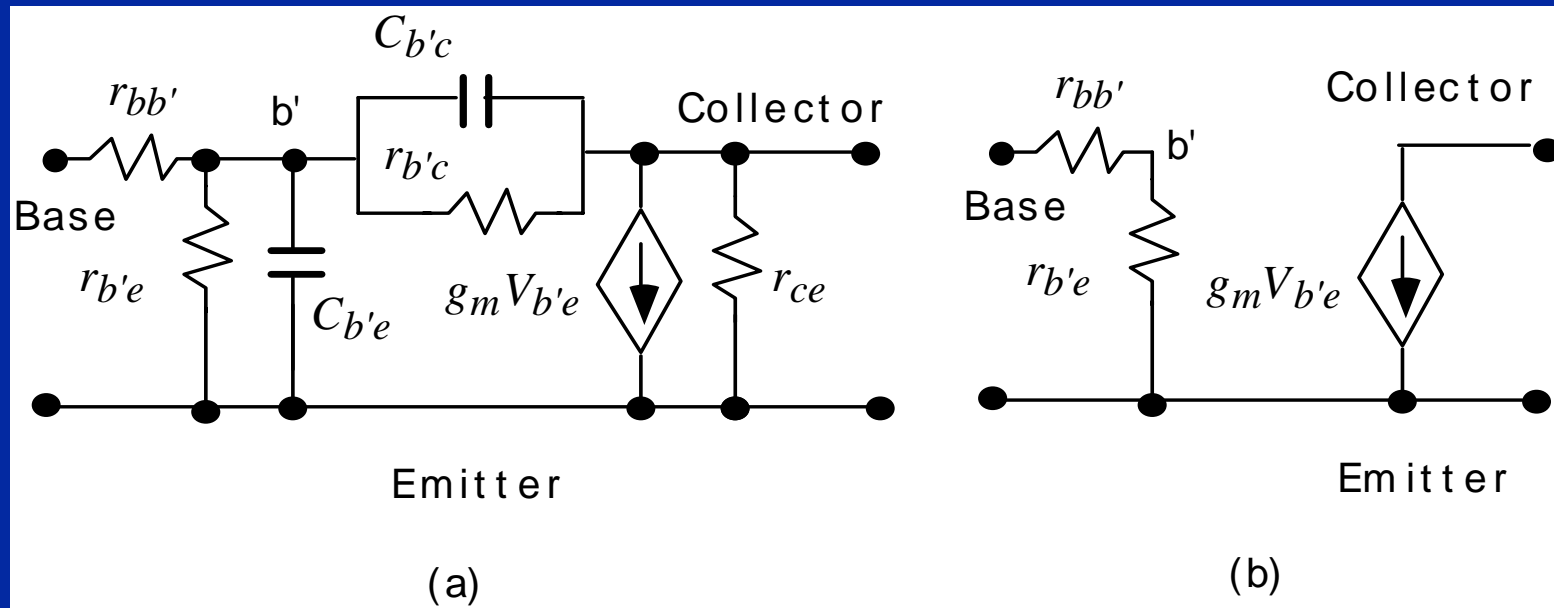
$$r_{b'c} = \frac{\partial V_{cb'}}{\partial I_b} \approx \frac{\partial V_{ce}}{\partial I_c / \beta_N} = \beta_N r_{ce} = \frac{\beta_N |V_A|}{g_m V_{th}}$$

$$C_{b'e} = C_{de} + C_{dife} = C_{de}(V_{b'e}) + g_m F \tau_F$$

$$C_{b'c} = C_{dc} + C_{difc} = C_{dc}(V_{b'c}) + g_m R \tau_R$$

From T. A. Fjeldly, T. Ytterdal, M. S. Shur, *Introduction to Device Modeling and Circuit Simulation*, Wiley, New York, 1998

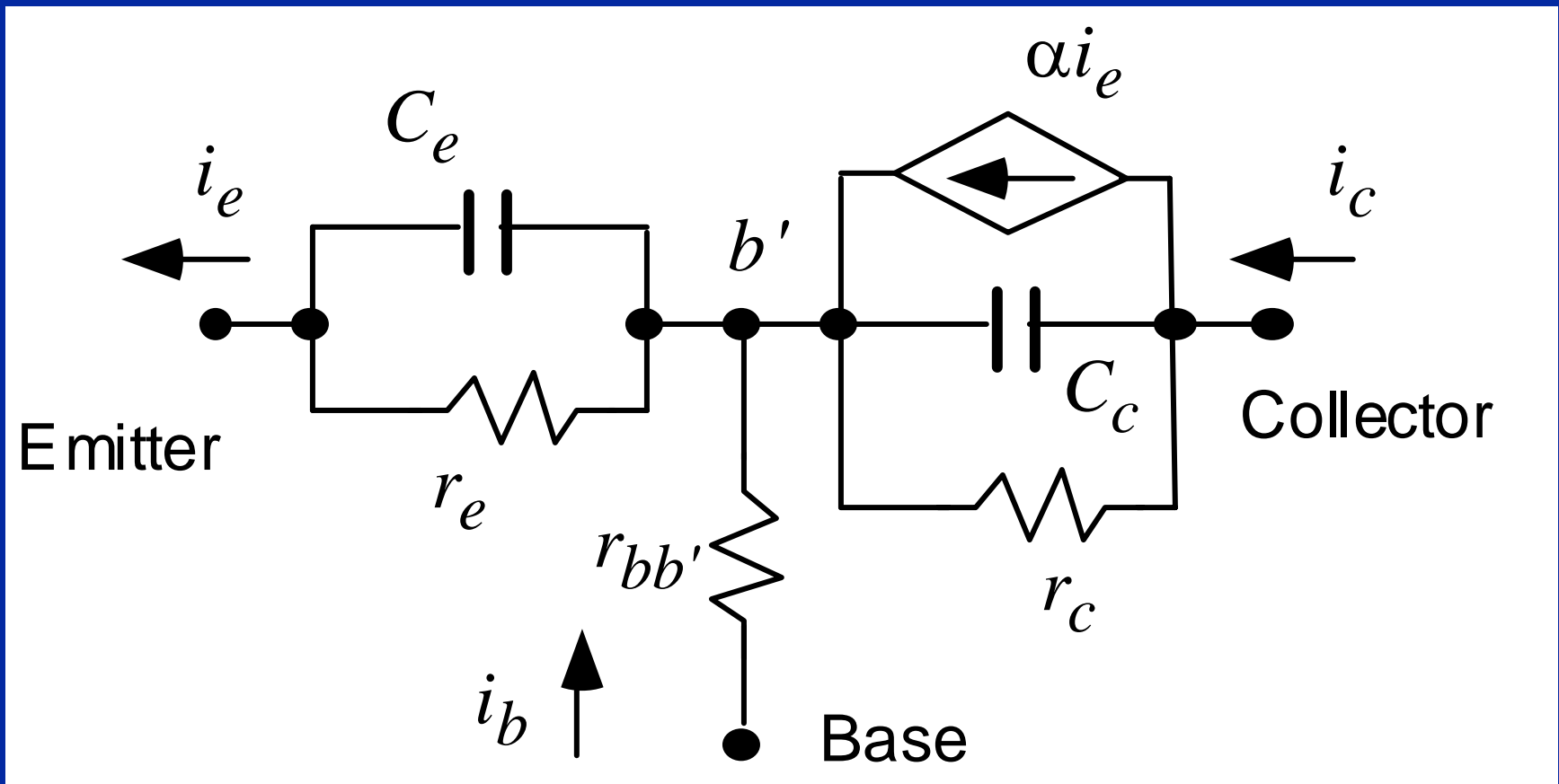
Full (a) and simplified (b) hybrid- π equivalent circuit



$$g_m = \frac{\partial I_c}{\partial V_{b'e}} = \alpha \frac{\partial I_e}{\partial V_{b'e}} \approx \frac{\alpha}{r_e} \approx \frac{I_c}{V_{th}}$$

$$r_{b'e} \approx \frac{i_c}{i_b} \frac{1}{g_m} \approx \frac{h_{fe}}{g_m} = \frac{\beta}{g_m}$$

T equivalent circuit



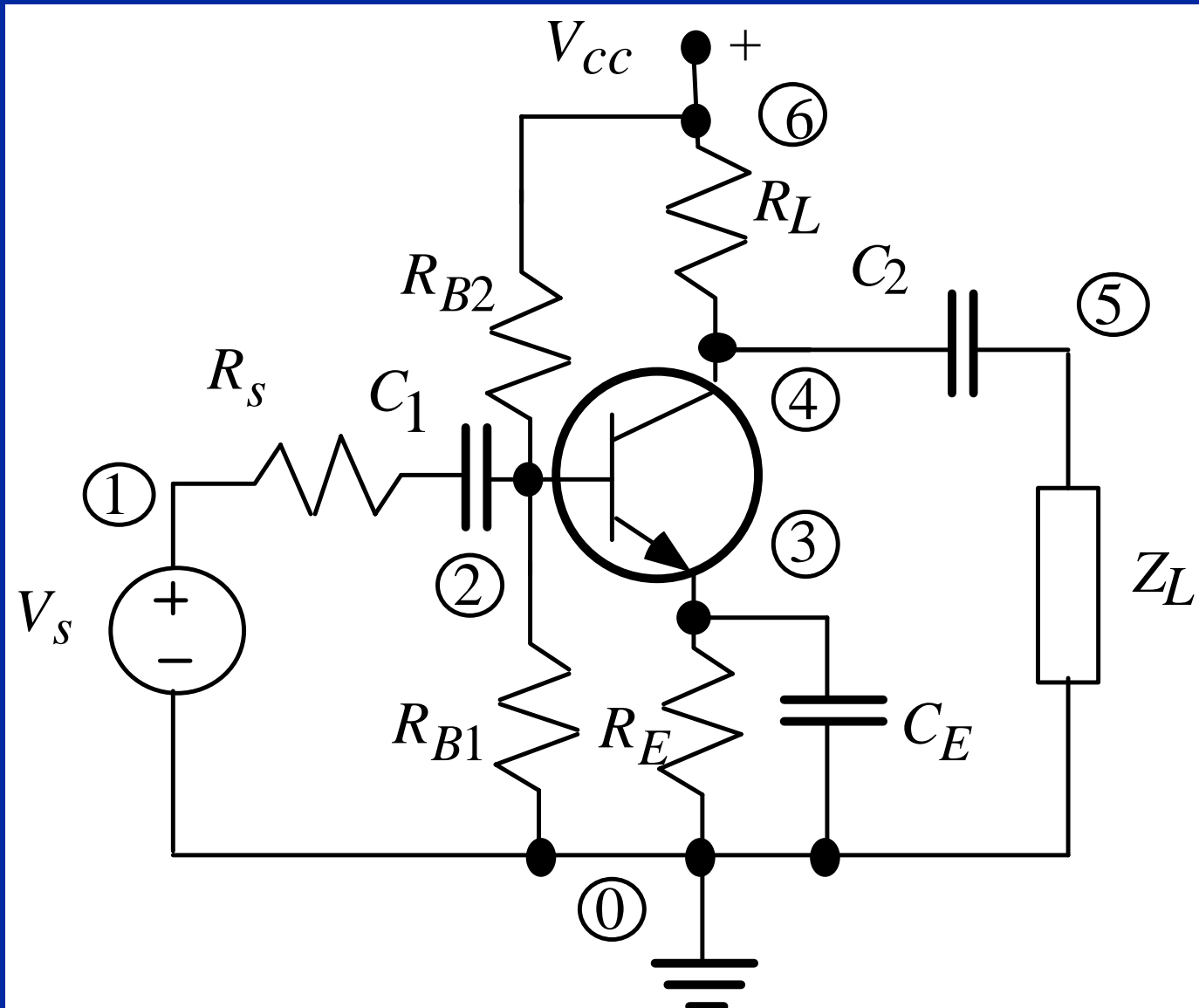
π , T and h

Parameter	Expressed through h -parameters for common-emitter configuration	Expressed through h -parameters for common-base configuration
r_e	h_{re} / h_{oe}	$h_{ib} - h_{rb} (1 + h_{fb}) / h_{ob}$
$r_{bb'}$	$h_{ie} - h_{re} (1 + h_{fe}) / h_{oe}$	h_{rb} / h_{ob}
r_c	$(1 + h_{fe}) / h_{oe}$	$(1 - h_{rb}) / h_{ob}$
α	$h_{fe} / (1 + h_{fe})$	$-h_{fb}$
β	h_{fe}	$-h_{fb} / (1 + h_{fb})$

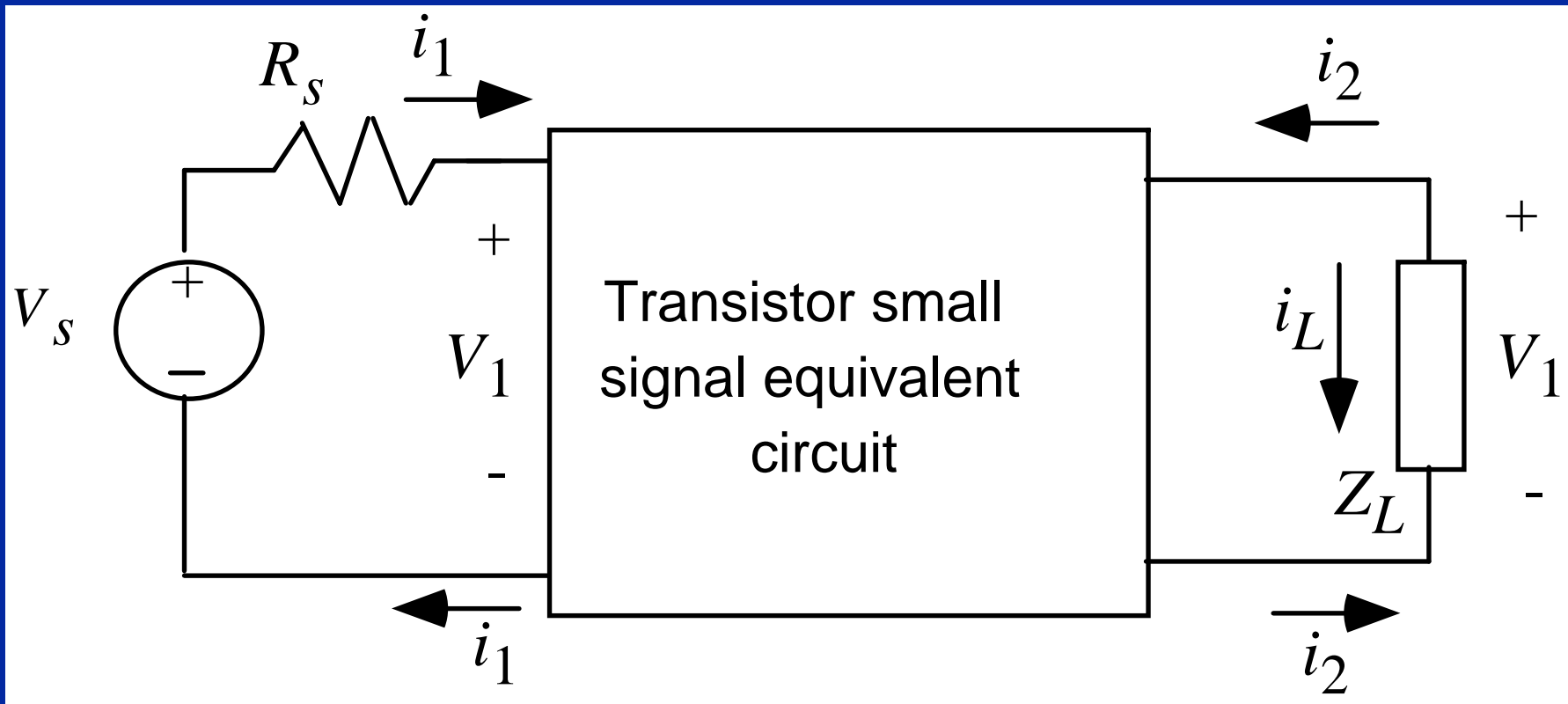
h -parameter	Relation to parameters of hybrid π -equivalent circuit
h_{oe}	$1 / (r_{b'c} + r_{b'e}) + 1 / r_{ce} + g_m r_{b'e} / (r_{b'c} + r_{b'e})$
h_{ie}	$r_{bb'} + r_{b'e} r_{b'c} / (r_{b'e} + r_{b'c})$
h_{fe}	$g_m r_{b'e} r_{b'c} / (r_{b'c} + r_{b'e})$

Important relationships

Definition of h -parameters	$v_1 = h_{11}i_1 + h_{12}v_2$	$i_2 = h_{21}i_1 + h_{22}v_2$
Dynamic resistance of the forward-biased emitter-base junction	$r_e = \frac{\partial V_{b'e}}{\partial I_e} \approx \frac{V_{th}}{I_e}$	
Transconductance	$g_m = \frac{\partial I_c}{\partial V_{b'e}} = \alpha \frac{\partial I_e}{\partial V_{b'e}} \approx \frac{\alpha}{r_e} \approx \frac{I_c}{V_{th}}$	
Resistance in the hybrid-p equivalent circuit	$r_{b'e} \approx \frac{i_c}{i_b} \frac{1}{g_m} \approx \frac{h_{fe}}{g_m} = \frac{\beta}{g_m}$	



Midband



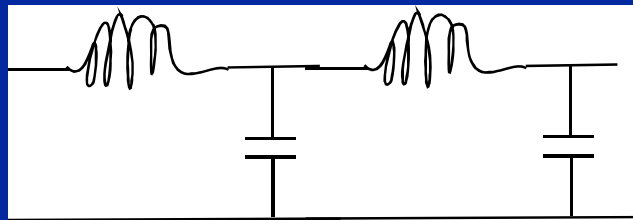
Gains and Impedances

	Definition	Relation to h -parameters
Input impedance	$Z_i = v_1/i_1$	$Z_i = h_i - h_f h_r / (h_o + Y_L)$
Output admittance	$Y_o = i_2/v_2$	$Y_o = h_o - h_f h_r / (h_i + R_s)$
Voltage gain	$A_v = v_2/v_1$	$A_v = A_i Z_L / Z_i$
Voltage gain	$A_{v_s} = v_2/v_s$	$A_{v_s} = A_v Z_i / (Z_i + R_s)$
Current gain	$A_i = i_L/i_1 = -i_2/i_1$	$A_i = h_f / (1 + h_o Z_L)$
Current gain	$A_{i_s} = -i_2/i_s$	$A_{i_s} = A_i R_s / (Z_i + R_s)$
Power gains	$A_p = A_v A_i, A_{p_s} = A_{v_s} A_{i_s}$	

Motorola 2N2219A transistor

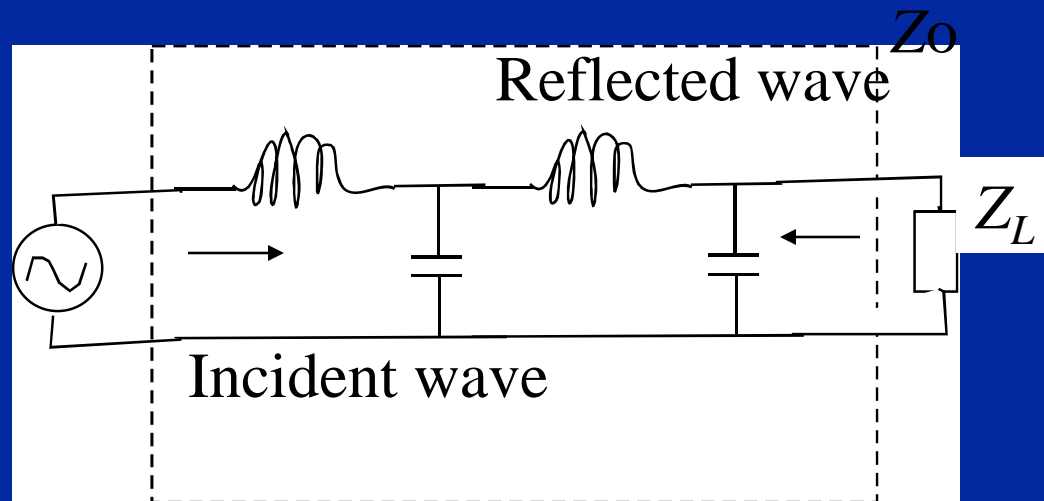
	Common emitter	Common base	Common collector
h_i (ohm)	1250	16.4	1250
h_r	4×10^{-4}	1.1×10^{-5}	1
h_f	75	-0.987	-76
h_o (μmho)	25	0.329	25
Z_i (kohm)	1.19	0.0164	146
Y_o (μmho)	15.77	0.334	24350
A_i	- 71.43	0.986	72.38
A_{is}	- 44.72 (33 dB)	0.977	0.978
A_v	- 119.76 (41.6 dB)	120.1	0.991
A_{vs}	- 44.72 (33 dB)	0.977	0.978

Lossless Transmission Line



$$Z_o = (L/C)^{1/2}$$

$$V = V_i + V_r$$

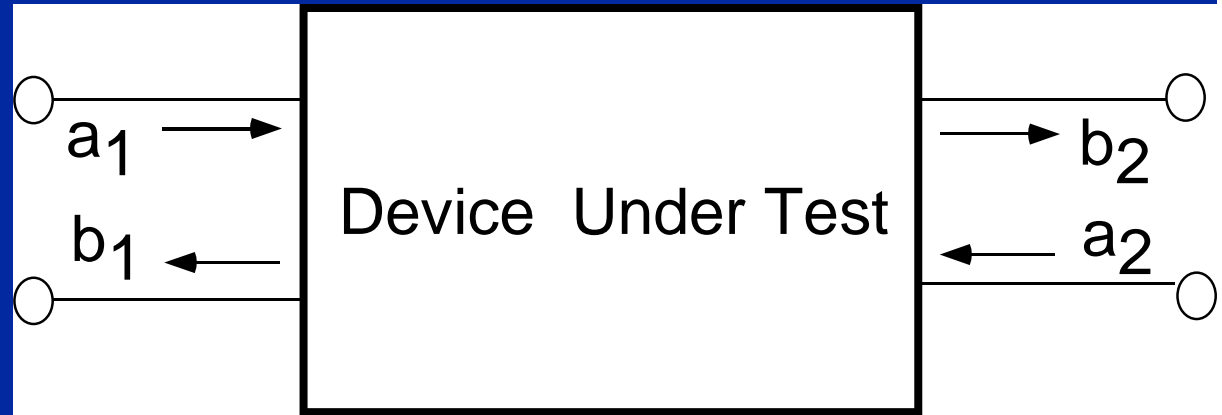


$$I = (V_i - V_r)/Z_o$$

$$\Gamma = (Z_L - Z_o)/(Z_L + Z_o)$$

$$V_{r1} = s_{11} V_{i1} + s_{12} V_{i2}$$

$$V_{r2} = s_{21} V_{i1} + s_{22} V_{i2}$$



Now divide both sides by $Z_o^{1/2}$

$$b_{r1} = s_{11} a_{i1} + s_{12} a_{i2}$$

$$b_{r2} = s_{21} a_{i1} + s_{22} a_{i2}$$

where

$$b_{r1} = V_{r1} / Z_o^{1/2}$$

$$b_{r2} = V_{r2} / Z_o^{1/2}$$

$$a_{r1} = V_{i1} / Z_o^{1/2}$$

$$a_{r2} = V_{i2} / Z_o^{1/2}$$

a_1^2 is the incident power
on port 1

b_1^2 is the reflected power
on port 1

Definitions

$$s_{11} = \left. \frac{b_1}{a_1} \right|_{a_2 = 0}$$

is called the **input reflection ratio**,

$$s_{12} = \left. \frac{b_1}{a_2} \right|_{a_1 = 0}$$

is called the **reverse transmission ratio**,

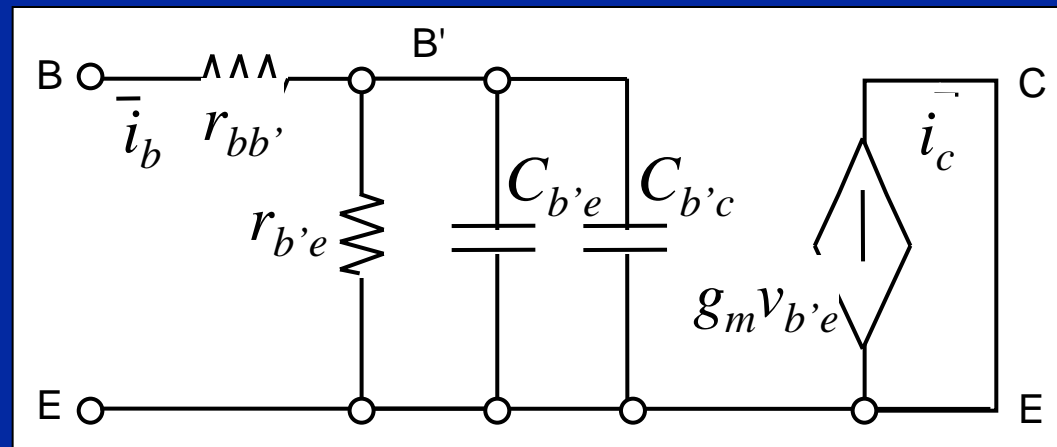
$$s_{21} = \left. \frac{b_2}{a_1} \right|_{a_2 = 0}$$

is called the **forward transmission ratio**, and

$$s_{22} = \left. \frac{b_2}{a_2} \right|_{a_1 = 0}$$

Cut-off Frequency f_T

Simplified hybrid- π equivalent:



Base ac input current:

$$i_b = v_{b'e} \left[\frac{1}{r_{b'e}} + j\omega(C_{b'e} + C_{b'c}) \right]$$

CE current gain versus ω :

$$\beta_\omega = \frac{i_c}{i_b} = \frac{\beta_N}{1 + j\omega/\omega_\beta}$$

From T. A. Fjeldly, T. Ytterdal, M. S. Shur, *Introduction to Device Modeling and Circuit Simulation*, Wiley, New York, 1998

Cut-off Frequency f_T (Cont.)

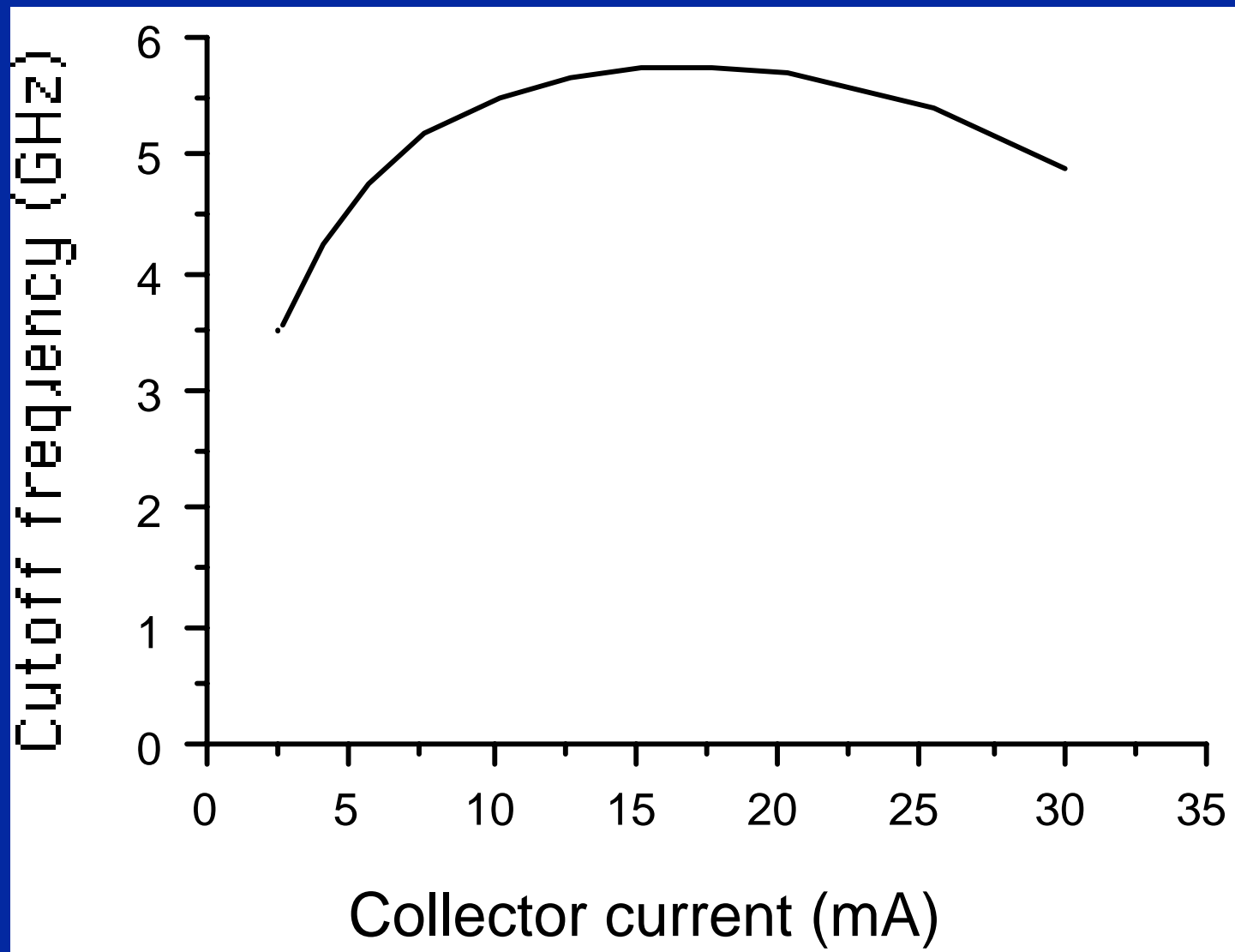
Beta cut-off frequency:

$$\omega_\beta = 2\pi f_\beta = \frac{1}{r_{b'e}(C_{b'e} + C_{b'c})} = \frac{g_m}{\beta_N(C_{b'e} + C_{b'c})}$$

Unity gain cut-off frequency ($|\beta_\omega| = 1$):

$$f_T = f_\beta \sqrt{\beta_N^2 - 1} \approx \frac{g_m}{2\pi(C_{b'e} + C_{b'c})}$$

From T. A. Fjeldly, T. Ytterdal, M. S. Shur, *Introduction to Device Modeling and Circuit Simulation*, Wiley, New York, 1998

f_T (GHz)

Cutoff Frequency for FET and BJT

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$g_m = \frac{\partial I_d}{\partial V_g} = q \frac{\partial n_s}{\partial V_g} v_s W \approx C_i \frac{v_s}{L}$$

$$f_T = 1 / 2\pi t_{tr}$$

where $t_{tr} = L/v_{eff}$ is the transit time of electrons in the channel. Assuming that v_{eff} , to be of the order of 5×10^4 m/s (which is about one half of the electron saturation velocity in Si) we obtain a characteristic transit time for a MOSFET, on the order of $t_{tr}(\text{ps}) \sim 20 L (\mu\text{m})$ and $f_T (\text{GHz}) \sim 8/L (\mu\text{m})$. In fact, the measured switching times may be quite a bit larger because the transistor response is slowed down by the parasitic and fringing capacitances, C_p , which add to the gate capacitance:

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd} + C_p)}$$

Compare with BJT:

$$f_T \approx \frac{g_m}{2\pi(C_e + C_{b'c})} = \frac{1}{2\pi\tau_{eff}}$$

f_{max}

The frequency at which the power gain of the transistor is equal to unity under optimum matching conditions for the input and output impedances is called the **maximum oscillation frequency**, f_{max} . Using a simplified π -equivalent circuit where we neglect $r_{b'c}$ and r_{ce} , we obtain

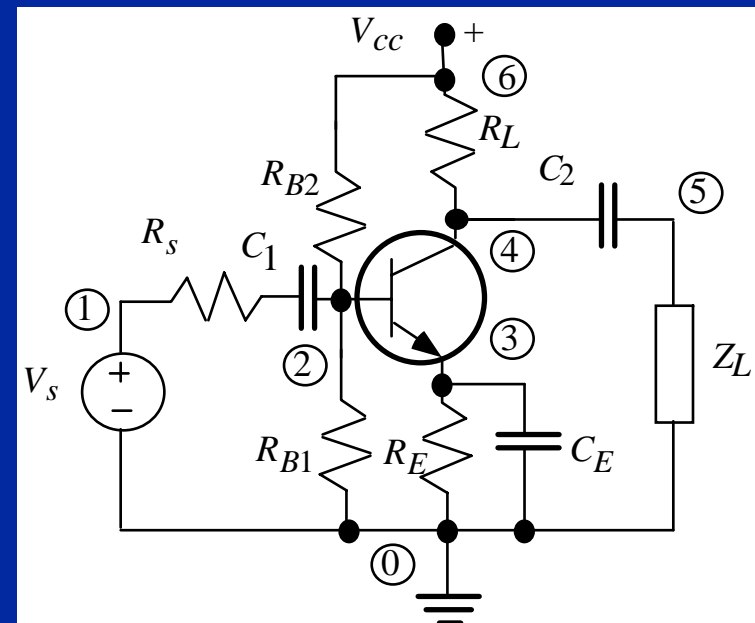
$$f_{max} = \sqrt{\frac{f_T}{8\pi r_{bb'} C_{b'c}}}$$

Solution

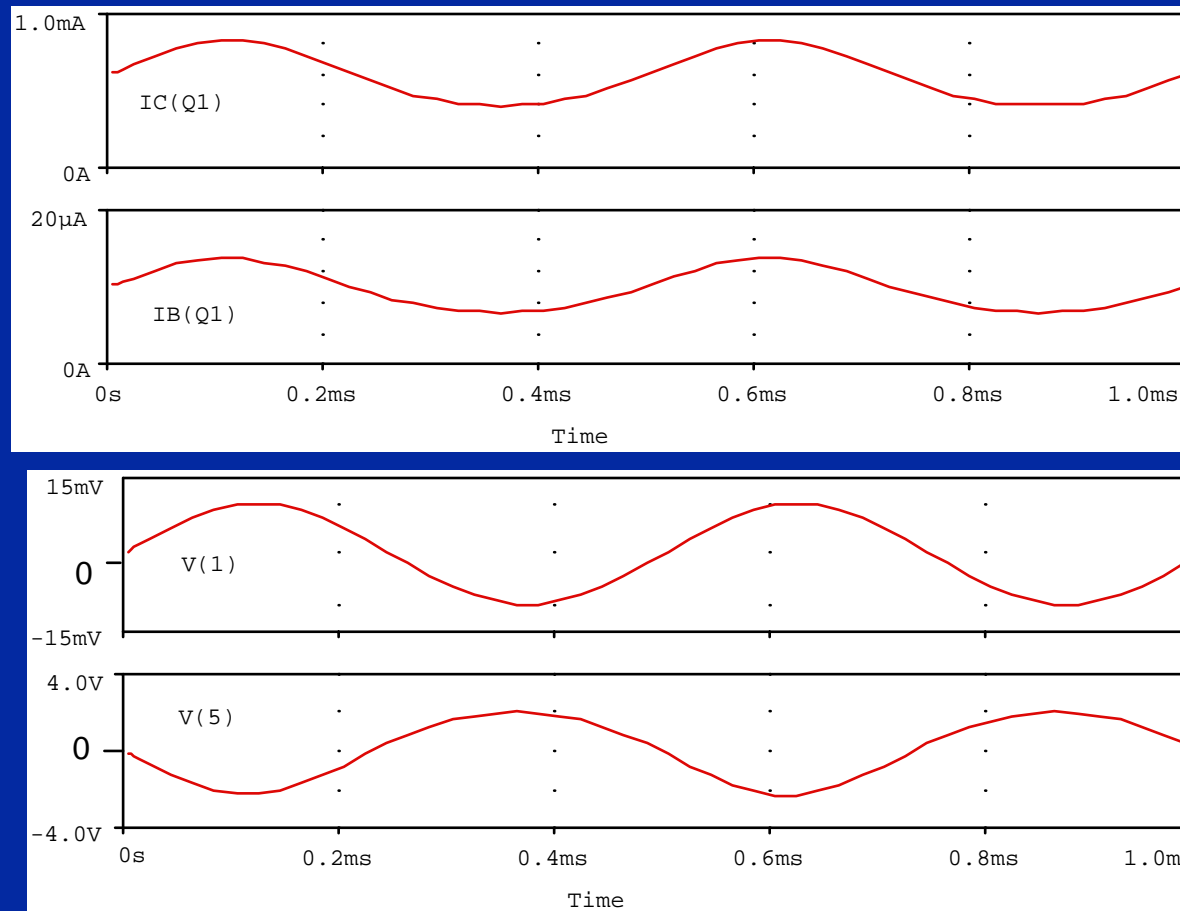
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Small Signal BJT Amplifier
Vcc 6 0 DC 15V
Vs 1 0 ac 1V sin(0 0.01V 2KHz)
C1 1 2 20u
RB1 6 2 120k
RB2 2 0 20k
RC 6 4 10k
RL 5 0 2MEG
RE 3 0 2K
CE 3 0 15u
C2 4 5 20u
Q1 4 2 3 BJT
.model BJT NPN (bf=60 rb=100 cjc=10p)
.ac dec 10 0.1 100Meg
.tran 0.1ms 1ms .005ms
.probe
.end

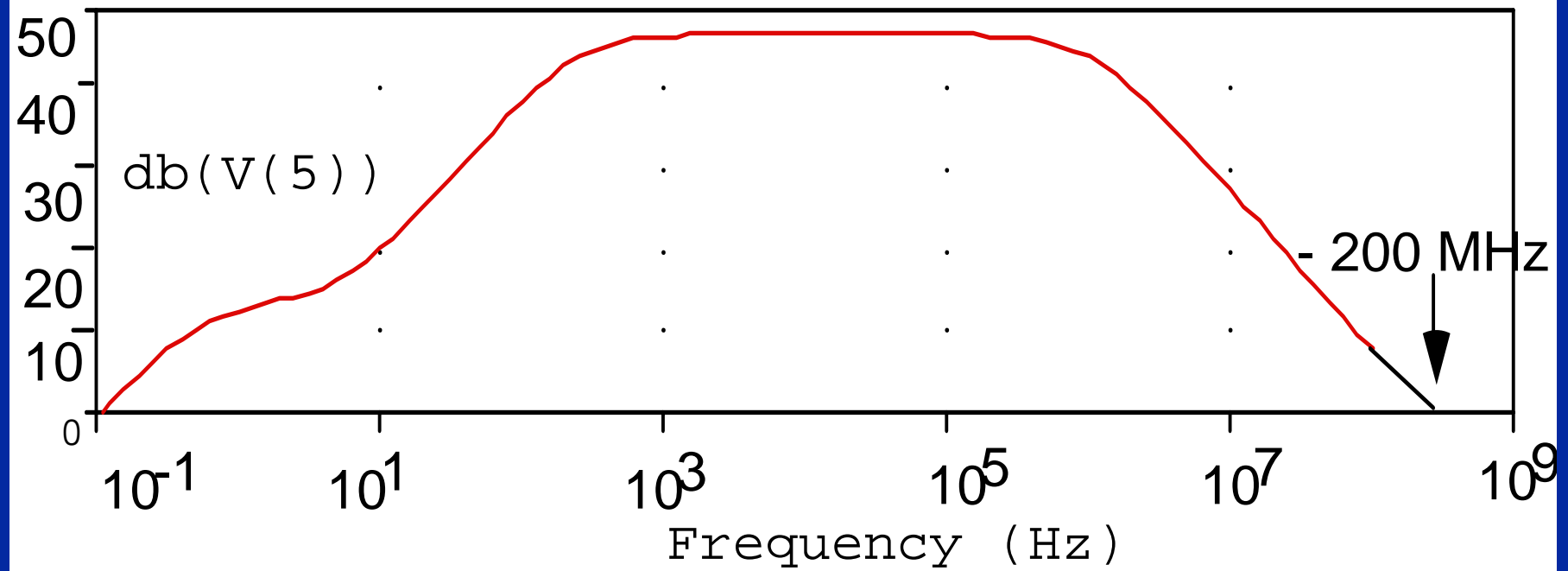
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Input and output current and voltages for one stage common emitter BJT amplifier



Voltage gain for one stage common emitter BJT amplifier



Summary

✖	
Common-base current gain	$\alpha_\omega = \alpha / (1 + j \omega / \omega_\alpha)$ where $\omega_\alpha = 2\pi f_\alpha = 1 / (C_e r_e)$
Common-emitter current gain	$\beta_\omega = \beta / (1 + j \omega / \omega_\beta)$ where $\omega_\beta = 2\pi f_\beta = g_{b'e} / (C_{b'e} + C_{b'c})$
Cutoff frequency	<p>Crude estimate: $f_T \approx \frac{g_m}{2\pi(C_e + C_{b'c})} \approx \frac{g_m}{2\pi C_e}$</p> <p>More accurate equation:</p> $f_T \approx \frac{1}{2\pi\tau_{eff}} \text{ where } \tau_{eff} = \tau_e + \tau_c + \tau_{cT},$ $\tau_e = (C_e + C_{b'c} + C_p) / g_m, \tau_{cT} \approx x_{dcb} / v_{sn}, \tau_c = r_{cs}$